Laser-Induced Sources for Magnetic Fields

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13. ABSTRACT

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PROBLEM STATUS

This is an interim report on a continuing problem.

AUTHORIZATION

LASER-INDUCED SOURCES FOR MAGNETIC FIELDS

INTRODUCTION

Large magnetic fields are produced when a powerful laser pulse is focused onto a solid target. The observation of these spontaneous magnetic fields in a laser-plasma (i.e., a laser-produced plasma) was reported in an earlier paper (1). In that paper (included here as App. A) we were able to account for major observed features of the fields by assuming a simple point source which allows thermal plasma energy to be converted into magnetic field energy. We did not discuss the conditions, due to laser radiation, which are responsible for these thermal sources. These conditions are discussed here in some detail. For clarity and completeness, the basic equation is derived and the meaning of terms is pointed out.

BACKGROUND DISCUSSION

The generation of magnetic fields is associated with the large pressure and temperature gradients which exist near the target where laser radiation is being absorbed. These gradients produce a solenoidal electric field which, as Faraday's law implies, is necessary for the generation of an initial magnetic field. The problem can be analyzed with the two-fluid (electron-ion) description of a collision-dominated neutral plasma.

The equation of motion for the electron fluid is*

$$nm \frac{dV_e}{dt} = -ne\left(E + \left(\frac{1}{c}\right)V_e \times B\right) - \nabla \cdot P^e + C . \qquad (1)$$

The force density $nm(dV_e/dt)$ on an electron fluid element is due to the averaged electromagnetic fields E and B, to the pressure P^e of adjacent electrons, and to collisions C with the ions given by

$$C = ne(R \cdot J + T \cdot \nabla T) \qquad (2)$$

where

$$J = ne(V_i - V_e) .$$

The collisional force C contains the Joule or resistive drag term proportional to the electric current density J, and a thermoelectric force term proportional to the temperature

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*In this report, tensors of rank 2 are denoted by boldface sans serif and Roman types.

gradient ∇T. The electrical resistivity is denoted by R and the thermoelectric power by T.

It is assumed that changes are sufficiently slow* so that one can ignore the electron inertia term $nm\, dV_e/dt$. Also, a scalar resistivity $R_{ij} = \rho\delta_{ij} = \sigma^{-1}\delta_{ij}$ is used, where σ is the electrical conductivity. This gives the expression

$$E = \left(\frac{J}{\sigma}\right) - \left(\frac{1}{c}\right)V_e \times B + \left(T \cdot \nabla T - \frac{1}{ne}\nabla \cdot P^e\right) \tag{3}$$

for the electric field.

One can obtain an equation describing the development of the magnetic field by taking the curl of Eq. (3) and combining the result with Maxwell's equations:

$$\frac{\partial B}{\partial t} = \nabla \times (V_e \times B) + \left(\frac{c^2}{4\pi\sigma}\right)\nabla^2 B + S \tag{4}$$

where

$$S = -c\nabla \times \left(T \cdot \nabla T - \left(\frac{1}{ne}\right)\nabla \cdot P^e\right). \tag{5}$$

Equation (4) shows how the magnetic field changes due to convection, diffusion†, and field generation. If only the first term were present on the right-hand side of Eq. (4), the magnetic field would be convected or carried with the electron fluid. The second term describes diffusion of the magnetic field and represents the eventual conversion of field energy to internal energy through Joule heating. The magnetic field changes due to diffusion are small compared to those resulting from convection when the magnetic Reynold's number R is somewhat greater than one. This was the case, at early times, in Ref. 1. The magnetic Reynold's number is given by $R = VL/D$ where $D = c^2/4\pi\sigma$ is the magnetic diffusivity and V and L are, respectively, average velocities and lengths characteristic to the flow.

The last term in Eq. (4) represents sources for the generation of a magnetic field and is of primary interest here. Solenoidal electric fields associated with either a pressure gradient or, through the thermoelectric power, a temperature gradient, are responsible for these sources.

Taking the inner or dot product of J with Eq. (3), and ignoring electron inertia ($m_e = 0$) so that $V_e = V - J/ne$, gives the rate per unit volume at which particle energy is converted to electromagnetic field energy as

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*Fast time changes (oscillations), due to quantities varying with the frequency of the electromagnetic field, are considered later in this report. As a result of these oscillations, radiation pressure becomes important and, in fact, should be added to the electron pressure.

†The displacement current can be ignored for the nonrelativistic velocities assumed here. In general, however, one must include the displacement current and, thus, allow (as $\sigma \to 0$) for electromagnetic wave propagation by replacing ∇^2 with $\nabla^2 - c^{-2}\partial^2/\partial t^2$ in Eq. (4).

$$-\mathbf{J}\cdot\mathbf{E} \;=\; \left(\frac{1}{c}\right)\mathbf{J}\cdot(\mathbf{V}\times\mathbf{B}) \;+\; \mathbf{J}\cdot\left[\left(\frac{1}{ne}\right)\nabla\cdot\mathbf{P}^{e} \;-\; \mathbf{T}\cdot\nabla T\right] \;-\; \frac{J^{2}}{\sigma} \tag{6}$$

where V is the flow or average velocity of the plasma.

The first two terms on the right-hand side of Eq. (6), when positive, represent, respectively, processes for converting plasma flow and thermal energy into electromagnetic field energy. The last term represents the irreversible dissipation of field energy into particle energy by Joule heating.

In astrophysics, the conversion of plasma flow energy to magnetic field energy is an important process called dynamo action, represented by the first term on the right-hand side in Eq. (6). The complicated motions (nonuniform rotation, turbulent and cyclonic convection, etc.) in stars, galaxies, and the earth's core can, through dynamo action, account for the magnetic fields of these bodies (2,3). Dynamo action, which depends on the electric field $\mathbf{V}\times\mathbf{B}/c$, requires an initial small magnetic field B, either originally present or, perhaps, generated by thermal sources.

In laser-plasmas, however, the most important energy source is thermal, represented by the second term on the right-hand side in Eq. (6). Large pressure and temperature gradients are produced when a powerful laser pulse is focused onto a solid target. Large gradients in electron pressure may be accompanied by large gradients in the radiation pressure. (The existence of nonthermal sources due directly to radiation pressure is discussed later.)

As expressed in Eq. (5), the thermal source function S depends on two tensor quantities—the electron pressure $\mathbf{P}^{e}$ and the thermoelectric power T. The meaning of the electron pressure tensor $\mathbf{P}^{e} = nm\langle\mathbf{V}'\mathbf{V}'\rangle$ should be clear. An average over the velocity distribution is denoted by $\langle\ \rangle$. The electron velocity V is written as the sum of a directed velocity $\langle\mathbf{V}\rangle = \mathbf{V}_{e}$ and a random velocity V'. If one ignores viscous effects but allows an anisotropy in P, then $P_{ii} = nkT_{i}$. An anisotropy due to the laser radiation is pointed out later.

As can be seen from Eq. (2), the thermoelectric power T is associated with electron-ion collisional effects. The tensor character of T is important when there is a magnetic field or the electron velocity distribution is anisotropic. The zero magnetic field (scalar) thermoelectric power, as evaluated by Braginskii (4a), is independent of density and temperature. This would show that there was no contribution to magnetic sources through the thermoelectric power. Under general conditions, however, *the thermoelectric power would contribute, but probably have a smaller effect than electron pressure.*

If one considers the case where P and T are scalars, one finds that ∇P and ∇T must be in different directions (a nonadiabatic condition) for the existence of a magnetic source ($S\neq0$). This necessary nonadiabatic condition is examined quantitatively later, but a general argument is presented here to show that ∇P and ∇T can have different directions in the presence of intense laser radiation.

Laser radiation has a greater effect on the heat flow than on the momentum flow or pressure. The heat flow due to nonlinear heat conduction can be very large (5), but it is limited by $nkTV_t$ where $V_t = \sqrt{2kT/m}$ is the electron thermal velocity. For the experimental conditions in Ref. 1, the ratio $I/nkTV_t$ of radiation to electron heat flow is of the order 1 and is much larger than the ratio $I/nkTc$ of radiation pressure to electron pressure. One would then expect the radiation to have a greater effect on the temperature gradient than on the pressure gradient so that they have different directions.

SPHERICAL SOURCE

The expansion of a laser plasma may have near spherical symmetry. However, the only magnetic field that can be generated with perfect spherical symmetry is a radial field, and this is ruled out by the absence of magnetic charge. Consider then, the special case where the only deviation from perfect spherical symmetry is that due to radiation energy flow, the radiation pressure being negligible. Thus, the plasma pressure $P = P(r)$ is a function of radius only but, since the normal component of the laser energy flux onto a spherical laser-plasma varies with the polar angle θ about the laser beam, we allow a variation of temperature T with angle θ. Thus $T = T(r) g(\theta)$, assuming azimuthal (ϕ) symmetry.
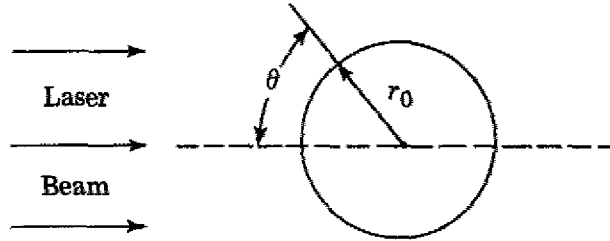
The source function, given by Eq. (5), due to a scalar electron pressure becomes $S = (ck/eP) \nabla T \times \nabla P f(t)$ where it is assumed that S has the time history $f(t)$ of the laser pulse. In spherical coordinates the source function (in an azimuthal direction) is then

$$S = \frac{ckT}{ePr} \frac{\partial P}{\partial r} g'(\theta) f(t) \tag{7}$$

where the polar variation $g'(\theta) = dg/d\theta$ is given. For a spherical laser-plasma of radius r_0, $g(\theta)$ contains a $\cos\theta$ factor for the projected area and a factor $\exp(-x^2/a^2)$ (for a Gaussian beam) for the variation of beam intensity with the distance $x = r_0 \sin\theta$ from the beam center. If, further, we assume a point source so that $\partial P/\partial r = P\delta(r)$, then S is of the form used in Ref. 1, except that a polar variation is supplied.

NECESSARY NONADIABATIC CONDITION

The role of laser radiation is now examined in more detail. It is first noted that nonadiabatic conditions are necessary for the existence of thermal magnetic sources. This problem can be analyzed by means of the electron heat equation (4b) given by

$$\dot{W}_H = \frac{nk}{\gamma-1}\frac{dT}{dt} - kT\frac{dn}{dt} . \tag{8}$$

This equation states that the rate per unit volume $\dot{W}_H$ at which energy is supplied to the electrons equals the rate $\dot{W}_I \equiv nk(dT/dt)/(\gamma-1)$ of change of internal energy plus the rate $\dot{W}_W \equiv -kT\,dn/dt$ at which work is done by the electrons. Adiabatic conditions exist when energy is supplied at such a slow rate ($\dot{W}_H \ll \dot{W}_I, \dot{W}_W$) that there is an approximate balance between the internal energy and work rates. We then have the relation $Tn^{1-\gamma} = $ const. between the temperature and density, implying that n and P are functions of T. Thus, the gradients are all in the same direction. Magnetic sources vanish under these adiabatic conditions.

However, if the laser energy is supplied to the electrons at a sufficiently high rate, then magnetic sources can exist in the resulting nonadiabatic state. The ratio $\dot{W}_H/\dot{W}_I$ can serve as a measure of nonadiabatic conditions: $\dot{W}_H$ is taken as the product of the absorption coefficient K (cm^{-1}) with the radiation intensity I (W/cm^2), and $\dot{W}_I$ is represented as nkT/τ_h where τ_h is an average electron heating time. Assuming that K is due to inverse bremsstrahlung absorption in the underdense region gives

$$\frac{\dot{W}_H}{\dot{W}_I} = \frac{\tau_h}{\tau_c}\frac{\left(\dfrac{I}{\bar{n}c}\right)}{n^*kT} \tag{9}$$

where τ_c is the electron-ion collision time (6a), $\bar{n}$ is the plasma index of refraction (≈ 1), and n^* is the critical density, i.e., the density (10^{21}cm^{-3} at 1.06μ) where the plasma frequency equals the laser frequency. Note that, since $\tau_h/\tau_c \gg 1$, nonadiabatic conditions can exist even when the ratio of radiation to electron pressure is rather small.

For the experimental conditions given in Ref. 1, the laser pulse width is large compared to the electron-ion equilibration time τ_{eq} (6b). Heating of the electrons should continue until the energy is shared with the ions. Thus, τ_h should be of the order of τ_{eq}. Making this assumption, and using the experimental values $I = 10^{12}W$/cm^2 and $kT = 100\ eV$ of Ref. 1, shows that $\dot{W}_H/\dot{W}_I$ is of the order one. Thus, the electrons were highly nonadiabatic, and magnetic sources did, of course, exist.

POLARIZATION AND ANISOTROPY EFFECTS

The only direction mentioned so far in characterizing the effect of laser radiation is the direction of propagation. Thus, in the calculation of a spherical source, there was azimuthal symmetry about this direction. The resulting magnetic field is in an azimuthal direction, as observed in Ref. 1. However, the laser radiation can be sufficiently intense ($\approx 10^{14}W$/cm^2) so that the usual linear polarization produces a corresponding anisotropy in the electron velocity distribution. The electron pressure and thermoelectric power are then anisotropic tensors. As a result, magnetic sources exist for any spatial variation (e.g., if all quantities vary exactly in a radial direction), and there is no azimuthal symmetry.

Electrons receive directed energy in the field of the focused, linearly polarized, laser beam. The electron velocities become randomized through many small-angle collisions.

(The cumulative effect of many remote collisions is more important in plasmas than the relatively few close collisions.) However, after a time τ_c (the collision time or 90° deflection time (6a)) energy is shared by the perpendicular component. One can thus equate the rate KI at which energy is deposited in the parallel component to the rate $nk(T_\| - T_\perp)/\tau_c$ at which energy is collisionally transferred from the parallel to the perpendicular component. $I(W/cm^2)$ is the radiant energy flux. Assuming that the absorption coefficient K is due to inverse bremsstrahlung in the underdense region gives an anisotropy

$$\eta \equiv \frac{T_\|}{T_\perp} - 1 = \frac{\left(\frac{I}{\tilde{n}c}\right)}{n^*kT} \tag{10}$$

This anisotropy becomes appreciable when the radiation pressure I/c is comparable to the particle pressure n^*kT. A comparison of Eqs. (9) and (10) shows that conditions can be highly nonadiabatic even though the laserinduced anisotropy, given by Eq. (10), is very small. This was the experimental situation in Ref. 1. The collision time τ_c does not appear in Eq. (10) since the τ_c in $nk(T_\| - T_\perp)/\tau_c$ cancels that in K. The result, Eq. (10), may thus be rather general, holding for effective collisions due to plasma turbulence.

When the electric field is in the x direction, the electron pressure can be written as

$$\mathbf{P}^e = nkT_\perp \begin{pmatrix} 1+\eta & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{11}$$

It is clear that $\nabla \cdot \mathbf{P}^e$ will have an appreciable solenoidal component, depending on the size of η. The loss of any azimuthal symmetry may be helpful in determining when one is in a regime where polarization effects are important. The author's estimate of the thermoelectric power tensor shows that it also has an anisotropy of the order η.

RADIATION PRESSURE

The role of radiation pressure is not so obvious. Clearly (due to their small mass) the electrons receive this initial momentum from the electromagnetic field. However, the radiation pressure exerted on the electrons is transmitted to the ions so that radiation pressure is usually thought of as being exerted on the neutral plasma. Despite this, it will be shown from a detailed look at radiation pressure effects that currents and magnetic fields are produced. One reported observation of currents was attributed, without proof, to radiation pressure (7), but those authors did not discuss any other (thermal) mechanisms for producing the currents. Two such mechanisms (nonadiabatic effects and anisotropy effects) were discussed earlier in the present report.

Radiation pressure is defined here as the time average $< >_t$, over many cycles, of the momentum flow tensor for the electromagnetic field, i.e., the negative of the Maxwell stress tensor for the radiation. (See App. B and C.) For a plane wave (where $\nabla \rightarrow i\mathbf{k}$ and $\partial/\partial t \rightarrow -i\omega$), Faraday's law gives

$$B = \frac{c}{\omega} k \times E = \bar{n} \hat{k} \times E \tag{12}$$

where $\bar{n} = \sqrt{\epsilon}$ is the index of refraction and $\hat{k} = k/|k|$. If AB is a product, of any tensor character, and if $(A,B) = (A_0, B_0)e^{-i\omega t}$, then $<\text{Re}(A)\ \text{Re}(B)>_t = 1/2\ \text{Re}(A_0 B_0^*)$. Thus

$$P^r = \frac{1}{8\pi}\left\{\frac{1}{2}\left[\text{Re}(E_0 \cdot E_0^*) + \text{Re}(B_0 \cdot B_0^*)\right] I - \left[\text{Re}(\epsilon E_0 E_0^*) + \text{Re}(B_0 B_0^*)\right]\right\} . \tag{13}$$

Plane and circularly polarized light are of particular interest. For plane polarized light, E_0 can be taken as $E_0(1,0,0)$ with B_0 equal to $\bar{n}E_0(0,1,0)$. For circularly polarized light of positive (upper sign) or negative (lower sign) helicity, $E_0 = (E_0/\sqrt{2})(1,\mp i,0)$ and $B_0 = (\bar{n}E_0/\sqrt{2})(\pm i,1,0)$. The factor $i = \sqrt{-1}$ means a $90°$ phase difference in time.

One finds that, *for both plane and circularly polarized light,* the intensity is

$$I \equiv \frac{c}{4\pi}\ |<E \times B>_t| = \frac{c\bar{n}}{8\pi}E_0^2 \tag{14}$$

and P^r is given by

$$P^r = \frac{I}{2\bar{n}c}\begin{pmatrix} 1-\epsilon & 0 & 0 \\ 0 & 1-\epsilon & 0 \\ 0 & 0 & 1+\epsilon \end{pmatrix} \tag{15}$$

where, for a plasma, the dielectric constant is $\epsilon = \bar{n}^2 = 1 - (\omega_p/\omega)^2 = 1 - (n/n^*)$ and n^* (10^{21}cm^{-3} at 1.06μ) is the critical density.

One can see from the anisotropy in P^r (i.e., $P_{||} \neq P_{\perp}$, where $P_{||}$ and $P_{\perp}$ are, respectively, the components of P^r parallel and perpendicular to the laser beam) that the force density ($\nabla \cdot P^r$) due to radiation pressure will, in general, have a solenoidal component.[†] It is shown in App. C, under simplified conditions, that $-(1/ne)\nabla \cdot P^r$ is a contribution to the electric field. Thus, radiation pressure produces a solenoidal electric field and must be included in the magnetic source term given by Eq. (5). The radiation pressure P^r is added to the electron pressure P^e in the source term. Radiation pressure results in a nonthermal source, i.e., radiation pressure can produce direct conversion between electromagnetic field energy and the energy of spontaneous magnetic fields.

It should be noted that radiation pressure effects represent not only nonthermal sources but also a nonthermal absorption of laser energy. Thus, part of the laser radiation is absorbed by direct conversion (no plasma heating) from laser field energy to magnetic field energy.

[†]For example, the anisotropy in P^r will result in a magnetic source $S = -c\nabla \times (-(\nabla \cdot P^r)/ne)$ even when all quantities vary precisely in a spherically radial direction. In this case, the source is in an azimuthal direction about the laser beam and has a magnitude given by

$$\frac{c \sin 2\theta}{er}\ \frac{\partial}{\partial r}\left[\frac{1}{n}(P_{\perp} - P_{||}) + \frac{r}{2n}\frac{\partial}{\partial r}(P_{\perp} - P_{||})\right] .$$

It should be possible to experimentally determine when radiation-pressure-induced magnetic sources are important. They exist only when the radiation is present and, for normal incidence onto a plane target, exhibit azimuthal symmetry about the laser beam. It is necessary to distinguish between sources due to a radiation-induced anisotropy in the electron velocity distribution and those due directly to radiation pressure. This can be accomplished by changing from plane polarized to circularly polarized light since this would change P^e but not P^r.

FIELDS IN THE ABSORPTION REGION

The maximum magnetic fields are produced in the region near the target while the laser radiation is being absorbed. Large gradients are produced in this absorption region as the plasma is ablated from the target. For the studies reported in Ref. 1, the gradients changed only slowly in time during the laser pulse. Let us assume, then, that the gradients are independent of time and have a magnitude equal to the absorption coefficient K.

The source function is an approximation to $(c/e)\nabla\times(\nabla P/n)$ when $|\nabla| \approx K$ and is represented here by

$$S = \frac{ckT}{e}K^2 \; . \tag{16}$$

In general, S must have an angular factor (e.g., $g'(\theta)$ in Eq. (7)) or anisotropy factor (e.g., η in Eq. (10)) reflecting its solenoidal character. This factor is taken as unity.

One can ignore diffusion ($R\gg1$) so that the time development of the field is determind by the growth due to sources and the decay due to the field being convected away with the expanding plasma. Thus

$$\frac{dB}{dt} + KVB = S \tag{17}$$

which has the solution

$$B = \frac{S}{KV}(1 - e^{-KVt}) \; . \tag{18}$$

For laser pulse widths large compared to the flow time $(KV)^{-1}$ out of the absorption region (as in Ref. 1), Eq. (18) gives

$$B = \frac{S}{KV} \tag{19}$$

which represents a balance between the effects of generation and convection.

Assuming that K is due to inverse bremsstrahlung in the underdense region and using experimental parameters gives $B \approx 0.1$ MG for the conditions in Ref. 1. This agrees

with the magnetic field obtained by extrapolating, with the known $1/r$ dependence, from 1 kG at 1 cm to the 100-μm focal radius.

For times short compared to the flow time out of the absorption region, the fields grow linearly with time as

$$B = St \ . \tag{20}$$

This case is of interest for subnanosecond laser pulses, but the assumption of steady-state gradients must be questioned for such pulses.

These considerations indicate that megagauss fields are produced in some experiments. Such fields would have a significant effect on much of the physics of laser-produced plasmas. The fields may affect not only the gross properties (e.g., plasma dynamics) of the plasma but also microscopic processes (e.g., momentum coupling, heating, and absorption of laser radiation).

CONCLUSION

The discussion presented here has concentrated on an important effect that intense laser radiation has on a plasma—the generation of large magnetic fields. The plasma conditions which allow this generation were examined. Sources associated with a scalar pressure or thermoelectric power require that the temperature and pressure gradients be in different directions. This nonadiabatic state results from the extremely high rate at which energy is delivered to the electrons. The electrons will also, in the presence of linearly polarized radiation of sufficient intensity, develop an anisotropic velocity distribution. Magnetic sources then exist for any spatial variation of the plasma parameters. It was shown that radiation pressure can directly (without plasma heating) contribute to magnetic field generation.

Much experimental work remains to be done. A full understanding will require detailed studies where the target and laser parameters (energy, pulse widths, and polarization) are varied. Measuring the fields in the absorption region is difficult, but optical techniques depending on the Faraday effect or the Zeeman effect may work.

There is a need for theoretical work in two areas. First, a full understanding of the generation processes requires some detailed theory. A quantitative theory is needed to evaluate the effect (through the temperature gradient) of thermoelectric power. Presumably, the thermoelectric contribution is smaller than that due to pressure gradients. At a high intensity ($\approx 10^{14} W/cm^2$), the fluid picture is inadequate, and plasma collective effects must be considered. Magnetic sources due to the direct effect of radiation pressure should be analyzed by a study of electron-ion interactions and plasma effects in the presence of intense radiation. The other area requiring attention is the physics of laser-plasmas in the presence (once generated) of large magnetic fields.

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APPENDIX A
(to NRL Report 7411)

Spontaneous Magnetic Fields in Laser-Produced Plasmas

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Spontaneously generated magnetic fields of the order of a kilogauss have been observed
in a laser-produced plasma, using a variety of targets and background pressures. The
generation of these magnetic fields is explained in terms of thermoelectric currents
associated with large temperature gradients near the target.

A dense, energetic plasma can be produced by focusing the pulse from a Q-switched laser onto a small solid target located in a background gas. We have observed the generation of large magnetic fields in such a laser plasma in the absence of any applied fields.

In our experiment, the target was located at the center of a large (12-in. i.d. × 54-in. long) Pyrex tube containing an ambient gas. A lens, located at the end of the tube, was used to focus the laser beam onto the target. A neodymium-doped glass laser was used to produce the laser plasma. It had an output of 60 J in 30 nsec with a beam diameter of 32 mm and a full-angle, half-power beam divergence of 200 μrad.

Magnetic probes were inserted radially through small side tubes near the target. The probe sup-̄ were in a plane perpendicular to the tube ̄ ̄ the vertical

fiber target. They could be oriented so as to record the time derivative $\dot{B}$ of either the axial (B_z) or azimuthal (B_φ) (with respect to the laser beam) component of the magnetic field. The probes consisted of small (diameter <1 mm) coils of wire. They were connected via 50-Ω coaxial signal cables to an oscilloscope which recorded $\dot{B}$. Several steps were taken to insure that the data was accurate and meaningful. The probes were calibrated for orientation and sensitivity in a fast probe calibrator. A probe was considered to be operating satisfactorily, in the experiment, only when signals were accurately reversed when the probe was rotated through 180°. This simple test[1] insures against spurious electrostatic signals. Replacement of probes was necessary, at times, since they were easily damaged by the intense laser plasma. The magnetic field was mapped on a shot-by-shot basis with

good reproducibility (~10%). The spontaneous
magnetic fields were observed with a variety of
probes. These included glass and epoxy-coated
probes, single and double probes, and probes
which were either open or closed to plasma pen-
etration into the interior of the coils. A large
(~1-cm-diam) probe, having a turns-area param-
eter over twenty times greater than the probes
normally used, was used to measure the fields
at large radii ($r \geq 4$ cm). The field intensities
in this range could not be accurately measured
on the small probes but, nevertheless, there
was agreement to within a factor of 2 or 3 for
the two probes.

The target in most of our studies was a 250-
μm-diam fiber of Lucite ($C_5H_8O_2$). This was ap-
proximately the diameter of the laser focal spot.
Lucite was chosen because it produced an ener-
getic laser plasma absorbing more than 95% of
the incident radiation, while the carbon and oxy-
gen provided opportunity for doing spectroscopic
studies. The studies indicate a peak electron
temperature in the laser plasma of about 100 eV.
Experiments were also made using aluminum and
silver surfaces as targets. The aluminum and
silver disks ($\frac{1}{16}$ in. thick × $\frac{1}{4}$ in. diam) were sup-
ported by an insulator. Most of the data were
taken in a nitrogen background. The background
gas was photoionized by energetic photons from
the laser plasma. This photoionization is com-
plete out to a radius of several millimeters and
then decreases as the inverse square of the radi-
us. Spectroscopic studies give an electron tem-
perature for the background plasma of about 3
eV and a degree of ionization of about 5% at $r=0$,
$z=-2$ cm for a Lucite target in a 200-mTorr
background of nitrogen[2] (see insert in Fig. 1).
The expanding laser plasma couples strongly to
the background. An interaction region or front
is observed,[3] by means of image converter pho-
tography and shadowgraphy, to travel outward
with an initial velocity of $(1-5) \times 10^7$ cm/sec.

Magnetic fields were observed as pulses which
propagated with the same velocity as the fronts
observed by optical means. The pulses became
diffuse, with a spatial extent of the order of the
radius at which they were observed. The magni-
tude of the spontaneous fields was insensitive to
background pressure. The fields in a 200- and
a 50-mTorr background of nitrogen were, within
experimental error, the same, and the fields in
a background of 6×10^{-1} mTorr of air (base pres-
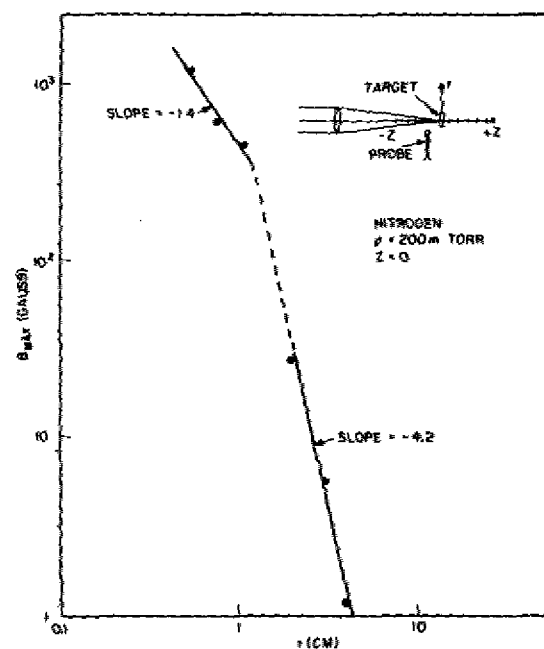sure) were only d-- -

FIG. 1. Radial variation of spontaneous fields.

azimuthal direction but, for a Lucite target,
showed a comparable axial component for $r \leq 1$
cm, $z=0$. The polarity of the azimuthal fields
implied conventional current flow in the direction
of the laser beam. Figure 1 shows the radial
variation of the maximum azimuthal field ob-
served in the midplane ($z=0$) for a Lucite target
in a 200-mTorr background of nitrogen with 10%
hydrogen added for diagnostics. For $r < 1$ cm,
$B \propto r^{-1.4}$. In this range, the front passes the
probe while the laser pulse is still incident on
the target. For $r > 1$ cm, $B \propto r^{-4.2}$. Here, the
laser pulse is over before the front reaches the
probe. A typical $\dot{B}$ oscillogram is shown in Fig.
2 for the same target and background as Fig. 1.
The probe was at $r=1$ cm, $z=0$. The maximum
field in this pulse is 450 G.

The Lucite laser plasma showed anisotropies
of the order of 2 in the velocity of early expan-
sion. A horizontal velocity twice that of the ver-
tical velocity was, presumably, due to the pres-
ence of fiber above and below the target region.
A preferred expansion velocity back toward the
laser was always present. If the laser pulse hit
the target at one side there was a perferred ex-
pansion to that side.
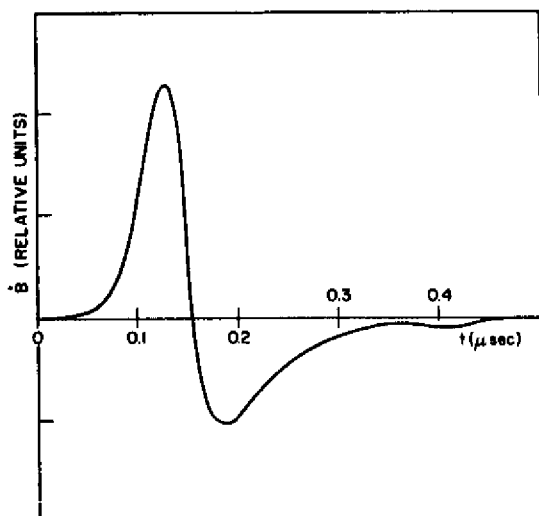
FIG. 2. Typical $\dot{B}$ oscillogram.

boundary-value problems. A more complete set of data with respect to axial position was taken in these experiments. Figure 3 gives the axial variations of the maximum fields in the pulse. The fields for an aluminum target were appreciably larger than for silver. This could be a result of the larger radiative energy loss from a silver target. The fields were, within experimental error, in an azimuthal direction about the incident laser beam for all probe positions. This is expected since the only anisotropy for a surface target is a higher expansion velocity back toward the laser.

The spontaneous generation of a magnetic field requires the presence of an initial solenoidal electric field. Near the focus there are large gradients in temperature. We consider the plasma and the target to form a thermoelectric junction with the laser focus as the hot junction. For the initial evolution of the laser-generated plasma and before any interaction occurs with the background plasma, one can apply a simple two-fluid model of a collision-dominated plasma since the plasma density near the focus is very large.

The generalized Ohm's law, neglecting electron inertia and ion pressure, is given by

$$\vec{J} = \sigma(\vec{E} + \vec{v}_e \times \vec{B}/c + \nabla P_e / n_e e - \overleftrightarrow{\alpha} \cdot \nabla T)$$

$$= (c/4\pi)\nabla \times \vec{B}, \qquad (1)$$

where $\overleftrightarrow{\alpha}$ is the plasma thermoelectric tensor; P_e, T, and n_e are, respectively, the electron pressure, temperature, and density; and the other
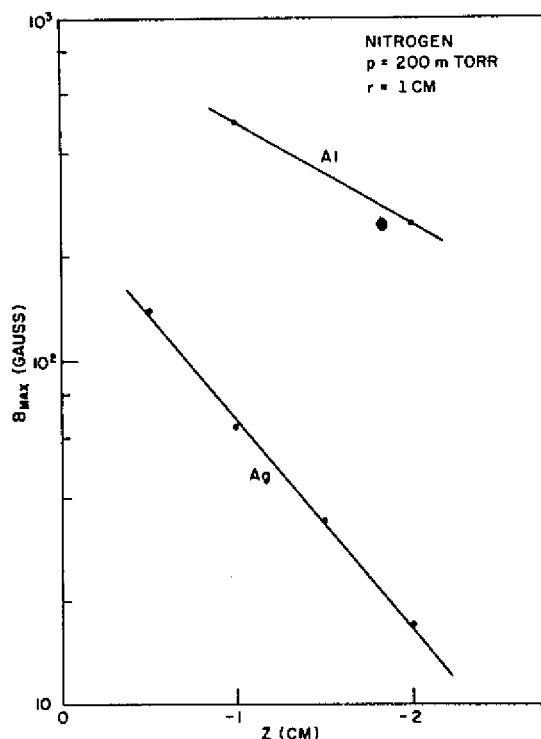

FIG. 3. Axial variation of spontaneous fields.

quantities have their conventional meaning. For a circuit moving with velocity $\vec{v}_e$, the rate of change of magnetic flux Φ over a surface S spanning it is given by

$$d\Phi/dt = \int_S [\partial \vec{B}/\partial t - \nabla \times (\vec{v}_e \times \vec{B})] \cdot d\vec{S}$$

$$= c \oint_C (\nabla P_e / n_e e + \overleftrightarrow{\alpha} \cdot \nabla T - \overleftrightarrow{\sigma}^{-1} \cdot \vec{J}) \cdot d\vec{S}. \quad (2)$$

The sources for the flux are the thermoelectric term $\overleftrightarrow{\alpha} \cdot \nabla T$ and, if $\nabla n \times \nabla T \neq 0$, the term with ∇P_e. These can be considered as an equivalent "battery" which drives the currents needed to produce the magnetic field. The thermoelectric contribution would vanish if $\overleftrightarrow{\alpha}$ were a scalar independent of position, i.e., if there were no discontinuities or junction contacts. From Eq. (1) and Faraday's law, assuming scalar conductivity, we obtain

$$\partial \vec{B}/\partial t = \nabla \times (\vec{v}_e \times \vec{B}) + (c^2/4\pi d)\nabla^2 \vec{B} + \vec{S}(\vec{r}, t), \quad (3)$$

where $\vec{S}$ is the source term.

Since we are interested in times and distances before any interaction with the background plasma occurs, the plasma conductivity will be given by the Spitzer formula.[4] The diffusion time for a length L is given by $\nu_{coll} \tau_{diff} = (L\omega_p/c)^2$. For the

plasma in this experiment, τ_{diff} exceeds the experimental times for distances larger than a few millimeters. Therefore neglecting the diffusion term in Eq. (3), the azimuthal component of the magnetic field is given in spherical coordinates (ρ, θ, φ) by

$$\partial B_\varphi/\partial t + \rho^{-1}\partial(\rho v_\rho B_\varphi)/\partial\rho = S(\vec{r}, t). \qquad (4)$$

In Eq. (4), we assume a spherically symmetric expansion of the laser plasma. The source term can be approximated at the focus of the laser by

$$S(\vec{r}, t) = (ckT_0/e)[\delta(\rho)/\rho]f(t). \qquad (5)$$

$S(\vec{r}, t)$ will, in general, depend on θ, but the explicit dependence has been ignored here. T_0 is the source electron temperature and $f(t)$ is the shape of the laser pulse in time. We can solve Eqs. (4) and (5) for B_φ, assuming that $v_\rho = v_0 U(t - \int_0^\rho d\rho/v_0)$:

$$B_\varphi(\rho, t) = (c/e)(kT_0/v_0\rho)f(t - \int_0^\rho d\rho/v_0), \qquad (6)$$

where U is the Heaviside unit step function.

Such a profile for the magnetic field appears to be consistent with the experimental observations at early times. The magnetic front propagates with the plasma expansion front as observed, and the duration is the approximate duration of the laser pulse. The maximum field falls off as $1/r$ which agrees approximately with experimental observations, Fig. 1, for $r < 1$ cm. The maximum field at any point r can be estimated from Eq. (6) if we take $T_0 \sim 100$ eV and $v_0 \sim 10^7$ cm/sec:

$$B_\varphi \approx (10^3/\rho)\ G,$$

which agrees in order of magnitude with the observations for a constant-velocity profile. The duration of the B pulse, at early times, is approximately that of the laser pulse, in agreement

with (6). The above analytical treatment aims at describing the main characteristics of the observations and has a number of shortcomings of detail. However, we think it covers the main effect both qualitatively and quantitatively.

At later times and longer distances, because of the momentum coupling of the expanding plasma with the background plasma, the simplified model presented above would need re-examination. Detailed study of the processes connected with the formation of a high-Mach-number shock are in progress.

In the course of writing the manuscript, our attention was called to the work of Korobkin and Serov[5] who observed a small spontaneous magnetic moment [(3-5)×10⁻³ Oe cm³] in the laser induced breakdown of a gas. Because of the meager data and sketchy description given, we cannot make a comparison between the two experiments.

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*On leave of absence from Cornell University.

[1]R. H. Lovberg, *Plasma Diagnostic Techniques*, edited by R. H. Huddlestone and S. L. Leonard (Academic, New York, 1965) p. 75.

[2]E. A. McLean, A. W. Ali, J. A. Stamper, and S. O. Dean, Bull. Amer. Phys. Soc. 15, 1411 (1970).

[3]S. O. Dean, E. A. McLean, and J. A. Stamper, Bull. Amer. Phys. Soc. 15, 1411 (1970).

[4]L. Spitzer, Jr., *Physics of Fully Ionized Gases* (Wiley, New York, 1962), 2nd ed., p. 138.

[5]V. V. Korobkin and R. V. Serov, Zh. Eksp. Teor. Fiz., Pis'ma Red. 4, 103 (1966) [JETP Lett. 4, 70 (1966)].

APPENDIX B
PLASMA CONTRIBUTION TO RADIATION PRESSURE

The effective radiation pressure (momentum flow of the electromagnetic field) in a medium must include the reaction of the medium to the radiation. This can be accomplished directly by including a dielectric constant and its variation with density. It can also be accomplihsed by considering the electron motion on a sufficiently small spatial scale that there is an oscillatory contribution to the electron momentum flow. This is included with the vacuum radiation pressure to obtain the effective radiation pressure in the medium.

The entire electron velocity distribution oscillates with the frequency ω of the electromagnetic field. A time average (for times small compared to the laser pulse width, but large compared to the oscillation period) is denoted by $< >_t$, while an average over the velocity distribution is denoted by $< >$. The velocity of an electron can be expressed as $\mathbf{v} = \mathbf{V} + \mathbf{v}' + \mathbf{v}_0$ where $\mathbf{V} = <\mathbf{v}>$ is the average velocity, $\mathbf{v}'$ is the random velocity, and $\mathbf{v}_0 = -ie\mathbf{E}/m\omega$ is the oscillating velocity (assuming that $\mathbf{E} = \mathbf{E}_0\, e^{-i\omega t}$). The electron momentum flow can thus be written as

$$\mathbf{D} \equiv nm <\mathbf{vv}> = nm\, \mathbf{VV} + nm <\mathbf{v}'\mathbf{v}'> + nm\, \mathbf{v}_0\mathbf{v}_0 \tag{B1}$$

when the random and oscillatory velocities are uncorrelated so that $<\mathbf{v}'> = 0$, $<\mathbf{v}_0>_t = 0$. The momentum flow thus consists of a macroscopic part $nm\, \mathbf{VV}$, a microscopic part or pressure $\mathbf{P} \equiv nm <\mathbf{v}'\mathbf{v}'>$, and an oscillatory part $nm\, \mathbf{v}_0\mathbf{v}_0$. The time-averaged momentum flow is

$$<\mathbf{D}>_t = nm\, \mathbf{VV} + nm <\mathbf{v}'\mathbf{v}'> + nm <\mathbf{v}_0\mathbf{v}_0>_t \tag{B2}$$

where $\mathbf{V}$ and $\mathbf{v}'$ are each assumed to change slowly over a wave period and to be represented by their time-averaged values.

The oscillatory momentum flow

$$nm <\mathbf{v}_0\mathbf{v}_0>_t = \frac{1}{2}nm\, \text{Re}(\mathbf{v}_0\mathbf{v}_0^*) = \frac{ne^2}{2m\omega^2}\, \text{Re}(\mathbf{E}_0\mathbf{E}_0^*) \tag{B3}$$

can be thought of as a plasma reaction contribution to the momentum flow or pressure of the electromagnetic field and may be included in this quantity.

A general form of this radiation pressure tensor is given by Pitaevskii* as

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*L. P. Pitaevskii, Sov. Phys. JETP, 12:1008 (1961).

$$P^r = \frac{1}{4\pi}\left\{\frac{1}{2}\left[\left(\epsilon - n\frac{d\epsilon}{dn}\right)E^2 + B^2\right]\mathbf{I} - (\epsilon EE + BB)\right\} \quad . \tag{B4}$$

Note, for a plasma in the underdense region, that $\epsilon = 1 - (n/n^*)$ so that $\epsilon - n(d\epsilon/dn)$ = 1. The radiation pressure used in this report can thus be seen to be the time average of Eq. (B4).

One could, instead, use the vacuum form of the radiation pressure tensor

$$P^r_v = \frac{1}{4\pi}\left[\frac{1}{2}(E^2 + B^2)\mathbf{I} - (EE + BB)\right] \tag{B5}$$

and add to this the oscillatory momentum flow contribution. The result, for a plasma in the underdense region, agrees with Eq. (B4).

It is clear from the above discussion that the oscillatory electron momentum flow is included, for this report, in the radiation pressure. The electron pressure thus does not include an oscillatory contribution. However, as noted on p. 5, the electron pressure is anisotropic (due to preferred heating along the E vector) and this leads to the generation of magnetic fields.

APPENDIX C
ELECTRIC FIELD DUE TO RADIATION PRESSURE

The various parameters characterizing a plasma undergo oscillations at the laser radiation frequency ω. As a result of these oscillations, the radiation pressure produces an electric field in the plasma. This electric field can be evaluated by considering that the various quantities in Eq. (3) for the electric field have an oscillating part and by taking a time average (over many oscillation periods) of the equation. For the neutral plasma, assumed in Eq. (3), the relevant field is $-(1/c)\mathbf{V}_e \times \mathbf{B}$. This field has two parts [since $\mathbf{V}_e = \mathbf{V}_i - (\mathbf{J}/ne)$]—the dynamo field, given by $-(1/c)\mathbf{V}_i \times \mathbf{B}$, and the Hall field given by

$$\mathbf{E}_H = \frac{1}{nec}\,\mathbf{J} \times \mathbf{B} \ . \tag{C1}$$

For slow variations with time, the Hall field does not result in a conversion of particle to field energy since $-\mathbf{E}_H \cdot \mathbf{J} = 0$ (see Eq. (6)). However, due to the high-frequency oscillations, the time-averaged Hall field contains the field $\mathbf{E}^r$ due to radiation pressure. The problem can be analyzed by assuming that the total quantities $\mathbf{E}^t$, $\mathbf{B}^t$, and $\mathbf{J}^t$ are the sum of a slowly changing part $\mathbf{E}^s$, $\mathbf{B}^s$, and $\mathbf{J}^s$ and a part $\mathbf{E}$, $\mathbf{B}$, and $\mathbf{J}$ oscillating with the frequency of the radiation. A time average (over times small compared to the laser pulse width, but large compared to the oscillation period) is denoted by $< >_t$. Then, since $<\mathbf{J}>_t = <\mathbf{B}>_t = 0$,

$$<\mathbf{E}_H>_t = \left(\frac{1}{nec}\right)\mathbf{J}^s \times \mathbf{B}^s + \left(\frac{1}{nec}\right)<\mathbf{J} \times \mathbf{B}>_t \ . \tag{C2}$$

The $(1/nec)\mathbf{J}^s \times \mathbf{B}^s$ field is the usual slowly varying Hall field which, as noted earlier does not contribute to magnetic field generation. However, the other term $(1/nec)<\mathbf{J} \times \mathbf{B}>_t$ does contribute to magnetic field generation and, for a neutral plasma, is just the electric field $\mathbf{E}^r = -(1/ne)\nabla \cdot \mathbf{P}^r$ due to radiation pressure. Thus

$$\mathbf{E}^r = \frac{1}{nec}<\mathbf{J} \times \mathbf{B}>_t \ . \tag{C3}$$

This result, for a neutral plasma, follows from momentum conservation of the electromagnetic field, given by

$$\nabla \cdot \mathbf{P}^{em} + \frac{\partial \mathbf{g}}{\partial t} = -\frac{1}{c}\mathbf{J}^t \times \mathbf{B}^t \tag{C4}$$

where $\mathbf{P}^{em}$ is the momentum flow tensor for the electromagnetic field (the negative of the Maxwell stress tensor) and $\mathbf{g}$ is the momentum density of the field. Taking the time average of Eq. (C4), assuming that $<\partial \mathbf{g}/\partial t>_t$ is small (since radiative flows are stationary),

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and associating the part of $\nabla \cdot \mathbf{P}^{em}$ due to the slowly changing part of the fields with $-(1/c)\mathbf{J}^s \times \mathbf{B}^s$, gives

$$\nabla \cdot \mathbf{P}^r = -\frac{1}{c}\langle \mathbf{J} \times \mathbf{B}\rangle_t \ . \tag{C5}$$

The field $\mathbf{E}^r$ due to radiation pressure is $-1/ne$ times the force per unit volume $\nabla \cdot \mathbf{P}^r$ due to radiation pressure.

The radiation pressure electric field is evaluated here in a special case. The collision frequency ν and plasma frequency ω_p are assumed small compared to the oscillation frequency ω of the radiation. The various quantities oscillate at the frequency ω. In complex notation this can be written as $(\mathbf{E},\mathbf{B},\mathbf{V},\mathbf{J}) = (\mathbf{E}_0,\mathbf{B}_0,\mathbf{V}_0,\mathbf{J}_0)e^{-i\omega t}$ so that $\langle \mathrm{Re}(\mathbf{J})\times \mathrm{Re}(\mathbf{B})\rangle_t = 1/2\ \mathrm{Re}(\mathbf{J}_0 \times \mathbf{B}_0{}^*)$. The electron motion is assumed to be described by the linearized equation of motion $m\ \partial\mathbf{V}/\partial t = -e\mathbf{E} - m\nu\mathbf{V}$ where $\partial\mathbf{V}/\partial t = -i\omega\mathbf{V}$. One can thus write $\mathbf{J} = n(-e)\mathbf{V} = \sigma\mathbf{E}$ where $4\pi\sigma \approx (\omega_p/\omega)^2(\nu + i\omega)$. Then, using complex notation,

$$\mathbf{E}^r = \left(\frac{1}{nec}\right)\langle \mathrm{Re}(\mathbf{J})\times \mathrm{Re}(\mathbf{B})\rangle_t = \frac{\left(\frac{\omega_p}{\omega}\right)^2}{8\pi\ nec}\left[\mathrm{Re}(\nu + i\omega)\mathbf{E}_0 \times \mathbf{B}_0{}^*\right] \ . \tag{C6}$$

The laser beam is assumed to be linearly polarized along the x axis and to be propagating in the $+z$ direction. Thus $\mathbf{E}_0 = E_0(1,0,0)$, and $\mathbf{B}_0 = \bar{n}B_0(0,1,0)$ where $\bar{n} = \sqrt{1 - (\omega_p/\omega)^2}$ is the index of refraction. The resulting radiation pressure field is

$$\mathbf{E}^r = \left(\frac{KI}{nec}\right)\hat{\mathbf{k}} \tag{C7}$$

where the absorption coefficient $K = (\omega_p/\omega)^2\ \nu/c$ is the absorption coefficient in the underdense region and $I = c\bar{n}|E_0|^2/8\pi$ is the intensity (in W/cm^2) of the radiation. The direction of $\mathbf{E}^r$ is that of the incident radiation $\hat{\mathbf{k}} = \mathbf{k}/|\mathbf{k}|$.

The force per unit volume $\mathbf{f}^r = n(-e)\mathbf{E}^r$ due to radiation pressure is thus

$$\mathbf{f}^r = -\left(\frac{KI}{c}\right)\hat{\mathbf{k}} \ . \tag{C8}$$

This result is expected since the rate per unit volume at which energy is absorbed is given by KI. The fact that the radiation pressure force $\mathbf{f}^r$ is opposite to that of the incident radiation $\hat{\mathbf{k}}$ is, in itself, important. Hora[†] calls the radiation pressure force a nonlinear deconfining force and has discussed its effect on plasma motion. The derivation given here (assuming the linear response of a neutral plasma) shows that this force has a direction precisely opposite to that of the incident radiation. Under general conditions the direction could be somewhat difficult.

[†]H. Hora, Phys. Fluids 12:182 (1969).

The discussion presented in the text shows that the radiation pressure force has a solenoidal character. The discussion presented in this appendix shows that this force can be described in terms of an electric field produced by radiation pressure. Radiation pressure thus produces a solenoidal electric field and will, at sufficiently high intensity $(I \gtrless 10^{14}\ W/cm^2)$, play an important role in the conversion between particle and field energy.